International Journal of Theoretical Physics, Vol. 14, No. 6 (1975), pp. 379–384

Does Euclidean Geometry Imply Quantum Physics?

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Received 15 *November* 1974

Abstract

In a previous paper, it was proposed that the cosmological term in Einstein's field equations be huge. This proposal heuristically followed from the combination of Leibniz' principle, Einstein's general relativity, and the observational dominance of Euclidean geometry. This paper presents preliminary results of a treatment of the large Λ field equations which holds promise of yielding quantum wave mechanics with no additional assumptions.

1. *Introduction*

Einstein's field equation with the cosmological term may be written*

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -(8\pi\kappa/c^2)\, T_{\mu\nu} \tag{1}$$

where κ is Newton's universal gravitational constant and c is 3×10^8 m/sec. The Leibniz principle, combined with the principles of general relativity and with the observed dominance of Euclidean geometry in successful descriptions of natural phenomena, suggests that Λ be huge, and that the associated "vacuum" or "flat space-time" be very dense, as discussed in Nickerson (1975). In geometrodynamic units (Misner et al., 1973), i.e., with $\kappa = c = 1$, Λ and $T_{\mu\nu}$ have the same dimensions. They are both mass-energy densities. Desiring to minimize the number of arbitrary physical constants, one looks for some combination of the basic constants that has dimensions of mass-energy density, $(\text{length})^{-2}$ in geometrodynamics. The only desirable candidate that pops into (my) head is $1/\hbar$, or $1/\hbar$ multiplied by a constant of order unity. Now $\hbar$, in geometrodynamic units, is the Planck length squared, or about 10^{-69} m^2. Clearly, then, $1/\hbar$ is huge. It is certainly larger than the largest known or contemplated observable densities. For example, the density of a neutron star is of order 10^{-9} m^{-2}. Thus $1/\hbar$ is a suitable candidate for the large Λ. It is desirable, as well, because it suggests quantum connections. We therefore take Λ to be $1/\hbar$, a very large positive number. With Λ a very large

* This paper uses the positive-time sign convention of Adler et al. (1975).

positive number, the $\Lambda g_{\mu\nu}$ term in Eq. (1) dominates, and it would seem appropriate to write $g_{\mu\nu}$ and $T_{\mu\nu}$ as large constants plus small "fluctuations" or perturbations. With small fluctuations, one is justified in linearizing Eq. (1). In Sec. 2, we shall examine this wave equation in enough detail to see the promise it holds for reproducing Schrödinger-like quantum wave mechanics. The main results of the preliminary investigations of Sec. 2 are (1) a Schrödinger-like quantum wave equation for scalar fluctuations in the metric with wave solutions of wavelength $\hbar^{1/2}$ = Planck length $\approx 10^{-35}$ m, (2) an indication that linear Dirac-like quantum equations for free particles will come from more detailed analysis, and (3) an identification of the Schrödinger wave function Ψ of quantum mechanics with a slow varying (long wavelength) amplitude modulation of the metric fluctuation, this modulation being of symmetric tensor character.

2. *Indications of Quantum Wave Mechanics from a Large Positive* Λ

In this section we shall consider the heuristic implications of a large positive Λ in Eq. (1). These results are very preliminary, and will be only briefly indicated. We write

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon\gamma_{\mu\nu} \tag{2}$$

and

$$T_{\mu\nu} = T_{\mu\nu}^{(\mathrm{EUC})} + f_{\mu\nu} \tag{3}$$

Here, $T_{\mu\nu}^{(\mathrm{EUC})}$ is the energy-momentum stress tensor for the "vacuum" or "flat space-time," and $f_{\mu\nu}$ represents fluctuations from $T_{\mu\nu}^{(\mathrm{EUC})}$. The "EUC" stands for "Euclidean," the geometry which describes flat space-time. As in Nickerson (1976), we write in geometrodynamic units (Misner et al., 1973), i.e., with $\kappa = c = 1$,

$$T_{\mu\nu}^{(\mathrm{EUC})} = -\Lambda\eta_{\mu\nu}/8\pi \tag{4}$$

The $\eta_{\mu\nu}$ is the Lorentzian metric appropriate to Euclidean geometry:

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{5}$$

Now, in the spirit of Leibniz' principle (Nickerson, 1976), we write

$$f_{\mu\nu} = \mu(\epsilon\gamma_{\mu\nu}) \tag{6}$$

where μ is a scalar function of order much smaller than Λ. That is, we assume that fluctuations in the background stress-energy tensor differ from fluctuations in geometry by a scalar factor μ such that $|\mu/\Lambda| \ll 1$.

Substituting Eqs. (2), (3), and (4) into Eq. (1) and subtracting out the large $\Lambda\eta_{\mu\nu}$ terms, we have

$$G_{\mu\nu} + \Lambda\epsilon\gamma_{\mu\nu} = -8\pi f_{\mu\nu} \tag{7}$$

We now linearize these equations by the standard technique (Adler et al., 1975, Sec. 9.1). This linearization has strong justification here in that the Leibniz principle plus our experience of the dominance of Euclidean geometry has implied that $g_{\mu\nu}$ is rigorously $\eta_{\mu\nu}$ plus a very small term and likewise $f_{\mu\nu}$ is small (Nickerson, 1976). That is, we have sound justification for the linearization of $G_{\mu\nu}$. As results from here on are preliminary, we continue with just a qualitative discussion of the linearized equation. Now $G_{\mu\nu}$ linearized is, to first order in ϵ, a linear second order differential operator, operating on $\epsilon\gamma_{\mu\nu}$

$$G_{\mu\nu} \to \left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}\right)\epsilon\gamma_{\mu\nu} + \text{crossterms like} \left(\frac{\partial^2}{\partial x \partial y}\right)\epsilon\gamma_{\mu\nu} \tag{8}$$

Thus (7) becomes a linear second order wave equation in the $\gamma_{\mu\nu}$, driven by $-8\pi f_{\mu\nu}$:

$$L_G[\gamma_{\mu\nu}] + \Lambda\gamma_{\mu\nu} = -8\pi\mu\gamma_{\mu\nu} \tag{9}$$

Here $L_G[\gamma_{\mu\nu}]$ is the linearized $G_{\mu\nu}$, and we have used $f_{\mu\nu}$ written as $\epsilon\mu\gamma_{\mu\nu}$, Eq. (6). Immediately, the large Λ term is a problem, a problem, however, that promises to lead us inevitably into quantum mechanics. The Λ will dominate wherever and whenever it appears because it is so large. Thus, at each stage we try to "get rid of it." The easiest way to get rid of it seems to be with Schrödinger-like wave equations, by techniques familiar from elementary mathematical physics, as follows: The wave solutions to a wave equation have the property of being homogeneous. That is, we suspect that we can write $\gamma_{\mu\nu}$ in the form

$$\gamma_{\mu\nu} = s_{\mu\nu}\phi \tag{10}$$

where ϕ satisfies the wave equation

$$L_G[\phi] + \Lambda\phi = 0 \tag{11}$$

Indeed, one can find such ϕ's and $s_{\mu\nu}$'s, and by plugging back into (9) can then eliminate the $\Lambda\gamma_{\mu\nu}$ term, thus solving the first problem with Eq. (9). Note that the wave solutions to (11) are characterized by a wavelength-period parameter of order $\Lambda^{-1/2}$, i.e., a frequency of order $\Lambda^{1/2}$. Thus ϕ is a very high frequency, short period wave. The idea of identifying $\Lambda^{-1/2}$ with $\hbar^{1/2}$ (in geometrodynamic units) and of calling Eq. (11) the quantum wave equation for the "graviton" is strongly suggested. The period parameter is then just the Planck length: $\Lambda^{-1/2} = \hbar^{1/2} \approx 10^{-35}$ m; so one has a theoretical justification for the idea that vacuum fluctuations occur with characteristic "size" 10^{-35} m.

We have not yet, however, completely got rid of Λ. Substituting (10) back into (9) and using (11), we eliminate the $\Lambda\gamma_{\mu\nu}$ term, but are left still with $\Lambda^{1/2}$ terms, as follows. Consider a typical term of $L_G[\gamma_{\mu\nu}] = L_G[s_{\mu\nu}\phi]$, say $(s_{\mu\nu}\phi)_{|\alpha|\beta}$ where "$_{|\beta}$", following Adler et al. (1975), Sec. 9.1, means $\partial/\partial x^\beta$. This gives us four terms:

$$(s_{\mu\nu}\phi)_{|\alpha|\beta} = s_{\mu\nu|\alpha|\beta}\phi + s_{\mu\nu|\alpha}\phi_{|\beta} + s_{\mu\nu|\beta}\phi_{|\alpha} + s_{\mu\nu}\phi_{|\alpha|\beta} \tag{12}$$

Now, the last term in (12), according to Eq. (11), combines with other such terms in $L_G[\phi]$ to give $-\Lambda s_{\mu\nu}\phi = -\Lambda\gamma_{\mu\nu}$ and eliminate the big "problem" in (9). The first term in (12) will be small; but the middle terms are of order $\pm i\Lambda^{1/2}s_{\mu\nu|\beta}\phi$ since $\phi_{|\alpha} = \pm i\Lambda^{1/2}\phi$. Thus we now have big $\pm i\Lambda^{1/2}s_{\mu\nu|\beta}\phi$ terms, which dominate the equation and must be considered next. Equation (9) now has the form

$$(L_G[s_{\mu\nu}])\phi + \text{dominant terms like } (i\Lambda^{1/2}s_{\mu\nu|\alpha})\phi = -8\pi\mu s_{\mu\nu}\phi \tag{13}$$

We can divide through by ϕ to get an equation in the $s_{\mu\nu}$ only, and if we take μ to be $m_0\Lambda^{3/2}$, with m_0 the rest mass-energy of an "elementary" particle, and if we assume that $m_0\Lambda^{3/2}(s_{\mu\nu}\phi)$ is much bigger than $(L_G[s_{\mu\nu}])\phi$, then the $\Lambda^{1/2}s_{\mu\nu|\alpha}$ and the $-8\pi m_0\Lambda^{3/2}s_{\mu\nu}$ terms dominate, giving us a linearized quantum wave equation for a free elementary particle of rest mass m_0:

sum of terms like

$$[i\Lambda^{1/2}s_{\mu\nu|\alpha}] = -8\pi m_0\Lambda^{3/2}s_{\mu\nu} \tag{14a}$$

or sum of terms like

$$\left[\frac{i}{\Lambda}\frac{\partial(s_{\mu\nu})}{\partial x^\alpha}\right] = -8\pi m_0 s_{\mu\nu} \tag{14b}$$

This does, indeed, look quite like the quantum mechanical wave equation for a free particle of rest mass m_0, with $s_{\mu\nu}$ the wave function. Writing Λ as $1/\hbar$, we have "operators" like $i\hbar\partial/\partial x^\alpha$, and we even have the i. One obviously wants to choose the constant a in the identification $\Lambda = a/\hbar$ such that the standard quantum equations are reproduced but a will be of order unity, and its value must await detailed study of the theory proposed here. Equations (14) are for free particles, i.e., particles in otherwise "evacuated" (no fluctuations) space, because we are now left with

$$(L_G[s_{\mu\nu}])\phi = 0 \tag{15}$$

and one guesses this is for free space-time. Also, there are conditions on m_0 and μ which must be met for this development to be valid. First, in deriving (13), we have assumed that $\mu\gamma_{\mu\nu}(=f_{\mu\nu}/\epsilon)$ is negligible compared to $\Lambda\gamma_{\mu\nu}$, i.e., $|\mu| \ll |\Lambda|$, which means $m_0\Lambda^{3/2} \ll \Lambda$ or $m_0 \ll 1/\Lambda^{1/2} \sim 10^{-35}$ m $\sim 10^{-8}$ kg. As all particles described by quantum mechanics have mass much less than this, the assumption seems all right. Of course, we may interpret this as a prediction that quantum mechanical effects are not important for the overall motion of "free" bodies of mass 10^{-8} kg or more. We have also assumed that

$$m_0 \Lambda^{3/2} = \mu \gg L_G [s_{\mu\nu}]/s_{\mu\nu} \tag{16}$$

which puts another condition on the geometry ($s_{\mu\nu}$) for which this analysis is valid. An (unchecked) guess is that this is the "free particle in space-time" condition. Another idea suggested here is that the "volume" of elementary "particles" is of order $\Lambda^{-3/2}$.

Examining further the requirement of approximate numerical equality for $|\mu s_{\mu\nu}|$ and $i\Lambda^{1/2} s_{\mu\nu|\alpha}$, one sees that this requires

$$\mu \approx \Lambda^{1/2}/\lambda_{\text{QM}} \tag{17}$$

where λ_{QM} is a usual quantum wavelength of order 10^{-10} m for electrons to, say, 10^{-15} m for high energy γ-rays and mesons. The $s_{\mu\nu}/\lambda_{\text{QM}}$ comes from $s_{\mu\nu|\alpha}$ in Eq. (14), where we take $s_{\mu\nu}$ to be a wave function. Equation (17) predicts a fundamental "density" for the "core" of elementary "particles" of order $1/(\hbar^{1/2}\lambda_{\text{QM}}) \approx 10^{45}\ \text{m}^{-2}$ to $10^{50}\ \text{m}^{-2}$. Combining (17) with $m_0 = \mu/\Lambda^{3/2}$, we get the de Broglie relationship for (elementary?) particles traveling near 3×10^8 m/sec: $m_0 = 1/(\Lambda\lambda_{\text{QM}}) = \hbar/\lambda_{\text{QM}}$.

For completeness' sake, here is the proposal for the field equations of general relativity and quantum wave mechanics:

$$G_{\mu\nu} + (a/\hbar) g_{\mu\nu} = -8\pi T_{\mu\nu} \tag{18}$$

with a a positive constant of order unity and $\hbar$ = Planck's constant.

Note the structure now of the geometric fluctuation $\gamma_{\mu\nu}$. It is an amplitude modulated high frequency tensor wave. The high frequency carrier ϕ is a scalar wave of characteristic period $\Lambda^{-1/2}(\sim 10^{-35}$ m). The amplitude modulation, the "slow varying" part $s_{\mu\nu}$, is of tensor character and is interpreted here as the wave function Ψ of quantum mechanics. It may be that the scalar-tensor nature can be reversed. That is, it would be of interest to try to make the fast varying part a tensor, and the quantum wave function a scalar. The possibilities have been only touched on here. The work is very preliminary.

3. *Discussion*

Section 2 outlined a development which seems to show that quantum mechanics is implied by the large Λ theory proposed in Nickerson (1976). Recall that the epistemological-philosophical desirables leading to the large Λ theory had, a priori, nothing to do with quantum mechanics. We began with only three concepts: two philosophical desirables, the Leibniz principle and general relativity; and one observation, the dominant experimental observation of our experience, *viz*, the ubiquitous success of Euclidean geometry in the description of all physical phenomena. These three concepts seem to lead directly to the quantum description of the universe. Thus we seem to get to quantum physics from strictly classical principles.

The treatment by wave analysis presented here also suggests a "reason" for the great success of linear theories in physics: The large Λ term justifies a linear perturbation treatment of fluctuations from the high density vacuum, these fluctuations being the observables described by linear theories.

Future work should include treatment of the large Λ field equations by means of the quaternion factorization program of Sachs (1967-72) and Edmonds (1974). The program would be to factor the large Λ field equations into linear quaternion, i.e., spinor, factors, and then proceed to eliminate the large Λ, $\Lambda^{1/2}$, etc., terms with Dirac-like wave equations.

Acknowledgments

Credit for any valid connections or effective presentations made here is due the grace of God. Many fruitful discussions with Joseph M. Brown and with Joe Ferguson on wide-ranging subjects have been significant in the developments presented. The author also acknowledges an invaluable education in philosophy and epistemology gained through discussions with J. Keehley and Wallace Murphree.

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